THE PLAYGROUND

Welcome to the Playground. Playground rules are posted on page 31, except for the most important one: *Have fun!*

THE SANDBOX

In this section, we highlight problems that anyone can play with, regardless of mathematical background. But just because these problems are easy to approach doesn't mean that they are easy to solve!

Problem 358. Randy Cohen of the Harvard Business School and Daniel Glickman of Aquant Capital Management contributed the following probability question. Randy and Daniel are playing a variation of the game H-O-R-S-E, alternately shooting baskets. First, Randy takes a shot. If it goes in, then Daniel must take the same shot. If Daniel makes this shot, Daniel wins the game; otherwise, Randy wins. If Randy misses the initial shot, then the game starts anew, with Daniel shooting first.

Assume both players have equal ability on all shots and that they know the probability of success for any given shot. In **Horsing Around,** you must find the probability of the shot Randy should attempt to maximize his chance of winning. What is the maximum winning probability for Randy?

Problem 359. Call a collection of positive integers $a_1 < a_2 < \cdots < a_n$ *square friendly* if the set of consecutive differences $\{a_2 - a_1, a_3 - a_2, \ldots, a_n - a_{n-1}, a_n - a_1\}$ are all distinct perfect squares. (Note that we consider a_n and a_1 consecutive.) For example, the set 2, 3, 12, 28, 77, 102 is square-friendly. In **Square-Friendly,** prove that there are square-friendly sequences of length n for all $n \geq 2$.

THE ZIP-LINE

This section offers problems with connections to articles that appear in the magazine. Not all Zip-Line problems require you to read the corresponding article, but doing so can never hurt, of course.

Problem 360. In "The Bingo Paradox" (see page 18), Arthur Benjamin, Joseph Kisenwether, and Ben Weiss explain why horizontal rows occur more frequently than vertical columns on winning bingo cards. But a 1×5 horizontal row and a 5×1 vertical column are not the only connected shapes that use five boxes. In fact, there are seven different ways to partition the number 5: 5, 41, 32, 311, 221, 2111, 11111. See the *Ferrers diagrams* for each of these partitions in figure 1. While these are not the only connected shapes possible, in this problem, we ignore reflections, rotations, and other 5-box variations. For **Bingo Partition,** find the probability that you win after five numbers have been called for each of these seven shapes. (The probabilities for the partitions corresponding to a horizontal row and a vertical column are given in the article.) Which shape gives the highest probability? (For instance, one way to win in five turns with the partition 311 is the sequence of letters GGGIN.)

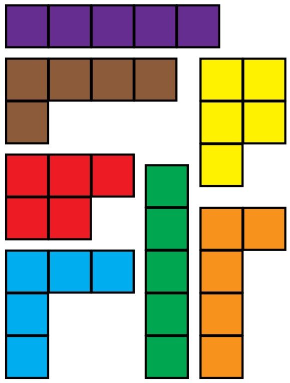

Figure 1. Ferrers diagrams for the seven partitions of 5: 5 (purple), 41 (brown), 32 (red), 311 (blue), 221 (yellow), 2111 (orange), and 11111 (green).

THE JUNGLE GYM

Any type of problem may appear in the Jungle Gym—climb on!

Problem 361. Brian Freidin and Peter McGrath of Brown University contributed **Binomial Identity.** Suppose n and k are positive integers with $k < n$. Prove that

$$\sum_{i=0}^{k} (-1)^{k-i} \binom{2n-1}{i} \binom{n-i-1}{n-k-1} = \binom{n+k-1}{k}.$$

THE CAROUSEL
OLDIES BUT GOODIES

In this section, we present an old problem that we like so much, we thought it deserved another go-round. Try this, but be careful—old equipment can be dangerous. Answers appear at the end of the column.

Show that $(1 + 2 + \cdots + n)^2 = 1^3 + 2^3 + \cdots + n^3$.