PROBLEMS AND SOLUTIONS

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Proposed problems and solutions should be sent in duplicate to the MONTHLY *problems address on the back of the title page. Proposed problems should never be under submission concurrently to more than one journal. Submitted solutions should arrive before February 29, 2012. Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

11593. *Proposed by Peter McGrath, Brown University, Providence, RI.* For positive integers k and n, let $T(n,k)$ be the $n \times n$ matrix with (i,j)-entry $((i-1)n+j)^k$. Prove that for $n > k+1$, $\det(T(n,k)) = 0$.

11594. *Proposed by Harm Derksen and Jeffrey Lagarias, University of Michigan, Ann Arbor, MI.* Let

$$G_n = \prod_{k=1}^{n} \left(\prod_{j=1}^{k-1} \frac{j}{k} \right),$$

and let $\overline{G}_n = 1/G_n$.

(**a**) Show that if n is an integer greater than 1, then $\overline{G}_n$ is an integer.

(**b**) Show that for each prime p, there are infinitely many n greater than 1 such that p does not divide $\overline{G}_n$.

11595. *Proposed by Victor K. Ohanyan, Yerevan, Armenia.* Let $P_1, \dots, P_n$ be the vertices of a convex n-gon in the plane. Let Q be a point in the interior of the n-gon, and let v be a vector in the plane. Let $\mathbf{r}_i$ denote the vector QP_i, with length r_i. Let Q_i be the (radian) measure of the angle between v and $\mathbf{r}_i$, and let F_i and Y_i be respectively the clockwise and counterclockwise angles into which the interior angle at P_i of the polygon is divided by QP_i. Show that

$$\sum_{i=1}^{n} \frac{1}{r_i} \sin(Q_i)(\cot F_i + \cot Y_i) = 0.$$

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