PROBLEMS

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PROPOSALS

To be considered for publication, solutions should be received by July 1, 2012.

1886. *Proposed by Jodi Gubernat and Tom Beatty, Florida Gulf Coast University, Fort Myers, FL.*

For which positive integers n is the function value

$$f(n) = \sum_{k=\lfloor n/2 \rfloor}^{n} \left(1 - \frac{2k}{n}\right)^2 \binom{n}{k}$$

an integer?

1887. *Proposed by Elias Lampakis, Kiparissia, Greece.*

Given a circle $\mathcal{C}$ with center O and radius r, and a point H such that $0 < OH < r$,

(a) Show that there are an infinite number of triangles inscribed in $\mathcal{C}$ with orthocenter H.

(b) Determine the set of points belonging to the interior of all triangles inscribed in $\mathcal{C}$ with orthocenter H.

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We invite readers to submit problems believed to be new and appealing to students and teachers of advanced undergraduate mathematics. Proposals must, in general, be accompanied by solutions and by any bibliographical information that will assist the editors and referees. A problem submitted as a Quickie should have an unexpected, succinct solution. Submitted problems should not be under consideration for publication elsewhere.

Solutions should be written in a style appropriate for this MAGAZINE.

Solutions and new proposals should be mailed to Bernardo M. Ábrego, Problems Editor, Department of Mathematics, California State University, Northridge, 18111 Nordhoff St, Northridge, CA 91330-8313, or mailed electronically (ideally as a LaTeX or pdf file) to mathmagproblems@csun.edu. All communications, written or electronic, should include **on each page** the reader's name, full address, and an e-mail address and/or FAX number.

1888. *Proposed by Alex Aguado, Duke University, Durham, NC.*

Let $A \subseteq X$ be a subset of a topological space, and let $N(A)$ denote the number of sets obtained from A by alternately taking closures and complements (in any order). It is well known that $N(A)$ is at most 14. However, exactly for which $r \leq 14$ is it possible to find A and X such that $N(A) = r$?

1889. *Proposed by Gary Gordon and Peter McGrath, Lafayette College, Easton, PA.*

For every positive integer k, consider the series

$$S_k = \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k}\right) - \left(\frac{1}{k+1} + \frac{1}{k+2} + \cdots + \frac{1}{2k}\right) + \left(\frac{1}{2k+1} + \frac{1}{2k+2} + \cdots + \frac{1}{3k}\right) - \left(\frac{1}{3k+1} + \frac{1}{3k+2} + \cdots + \frac{1}{4k}\right) + \cdots.$$

Thus $S_1 = \log 2$ and $S_2 = (\pi + 2\log 2)/4$.

(a) Prove that S_k converges for all k.

(b) Prove that

$$S_k = \int_0^1 \frac{x^k - 1}{(x^k+1)(x-1)}\,dx.$$

(c) Prove that the sequence $\{S_k\}$ is monotonically increasing and divergent.

1890. *Proposed by Erwin Just (Emeritus), Bronx Community College of the City University of New York, Bronx, NY.*

Let m and n be positive integers. Prove that there exist an integer k and a prime p such that $m \equiv k^2 + p \pmod{n}$.

Quickies

Answers to the Quickies are on page 68.

Q1017. *Proposed by Allan Berele and Jeffery Bergen, Department of Mathematics, DePaul University, Chicago, IL.*

Find a monic polynomial $f(x)$ with integer coefficients such that $f(x) = 0$ has no integer solutions but $f(x) \equiv 0 \pmod{p}$ has a solution for every prime p.

Q1018. *Proposed by Finbarr Holland, School of Mathematical Sciences, University College Cork, Cork, Ireland.*

Suppose $0 < \alpha \leq 1$. Prove that

$$e^x \leq \frac{1 + (1-\alpha)x}{1 - \alpha x}$$

for all $x \in [0, 1)$ if and only if $\alpha \geq 1/2$.