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Editor's Note: We publish the following memorial statement written by Margarita Tetlalmatzi-Montiel regarding her co-author, Jaime Cruz-Sampedro. "On November 3, 2015 the mathematical community lost a great friend with the passing of Jaime Cruz Sampedro. We knew Jaime as a person who supported our efforts to do important things and who kept open a wary eye to make sure we were living up to his high expectations. We were always the better for this and will now try to continue to bear in mind his standards. We owe him a lot and will miss him a lot."

An Alternative Approach to the Product Rule

The usual proof of the product rule in calculus involves adding and subtracting the same quantity, which can be unintuitive for students. Here is an alternative which uses the fact that fg arises as a cross term in $(f+g)^2$.

Using the power rule,

$$((f+g)^2)' = 2(f+g)(f'+g') = 2\left(ff' + gg' + f'g + fg'\right).$$

On the other hand, by expanding first and then differentiating,

$$((f+g)^2)' = \left(f^2 + g^2 + 2fg\right)' = 2\left(ff' + gg' + (fg)'\right).$$

Comparing the above expressions reveals the product rule

$$(fg)' = f'g + fg'.$$

One need not know the chain rule to carry out these calculations because it is easy to derive the formula for the derivative of the square of a function directly:

$$\begin{aligned}\left(u^2\right)'(x) = \lim_{h\to 0}\frac{u^2(x+h)-u^2(x)}{h} &= \lim_{h\to 0}\left(u(x+h)+u(x)\right)\frac{u(x+h)-u(x)}{h} \\ &= 2u(x)u'(x).\end{aligned}$$

—Submitted by Piotr Josevich

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